REDUCTION OF A HOSPITAL NETWORK AS A MULTIPLE CRITERIA OPTIMISATION PROBLEM

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Introduction

The health care systems in the former socialist countries of Central and Eastern Europe are uneconomical due to the capacity redundancy of hospitals [8]. There is an excess of beds for acute illnesses. According to WHO statistics for year 2001, the EU members (before May 2004) had in average 406 acute hospitals beds per 100,000 people compared with 631 in the Czech Republic and 674 in the Slovak Republic. Moreover, the distribution of hospital beds among medical specialities is improper. That is why reducing and restructuring the hospital network has been taking place. In the Slovak Republic, the process of restructuring was initiated in 2005, when the Order No 751/2004 Coll. Of the National Council of the SR about the minimal public health care facilities network had come into effect. Regarding hospitals, the Order defines the number of beds required for each health care specialisation. The Order was later modified and amended by the Order No 504/2007 Coll. Of the National Council of the SR (effective as of November 15 2007) which names the hospitals that the minimal network includes. Both of the Orders were openly criticised by the Health Policy Institute (HPI) [15]. The HPI has recommended the Government to define the accessibility and capacity standards which the health care providers have to guarantee. The Government should not specify which hospitals are supposed to be included in the minimal network. This is to be done by health insurance companies. They should decide which hospitals (or departments) will be financed by their money (or, better to say, by the insurance company clients´ money), in order to provide good quality and accessible health care services. This exactly is what is happening nowadays in our country. Since the beginning of the year 2008, two biggest health insurance companies in the Slovak Republic, namely the private company Dôvera and the state company Všeobecná zdravotná poisťovňa (VšZP) have been reducing their contracts with hospitals. Simultaneously, they have changed the structure of the ordered services. At the end of the year 2007 in the Slovak Republic there were 73 general hospitals which had at least one emergency department. In our study this hospital network is regarded as a complete network. In September 2008 8 hospitals had no contract with Dôvera, and 4 hospitals did not have a contract with VšZP.

The aim of this paper is to propose a mathematical model of a hospital network design which may serve the health insurance companies as a decision-supporting tool. Given a complete network of hospitals and the desired number of hospitals, the model decides (according to multiple criteria) which hospitals should be closed down, or where the open hospitals should be located. At the same time, it determines the allocation of customers to the open hospitals, defining the attraction zone of each hospital. The criteria comprise: (1) quality of the hospitals, (2) complexity, i.e. the number of emergency departments in the hospitals, (3) transportation accessibility, and (4) equitable distribution of the hospitals among citizens. Our goal is to find a solution which would represent the maximum utility across all of the objectives.

In the literature devoted to health management, there can be find econometric models for pricing hospitals [18] or for evaluating the effects of hospital closure on market [6]. These approaches are based on a model of patient's choice of the hospital. The choice depends on the distance to the hospital, quality of the hospital, and other characteristics of both the hospital and the patient. The application of these models in our region is limited as they are developed for urban areas with high density of population as well as hospitals where access is not an important issue.

On the other hand, operations research models of the health care facility location accentuate the patient's access to the facility. Patient's travelling costs related to reaching the facility is in most cases the only criterion for decision on facility opening or closure [5]. Only recently several works have appeared, taking into consideration a multi-objective character of the location-allocation problems in the health care sector (e.g. [10] or [16]). Nevertheless, also these approaches use traditional measures of the system design quality such as distance between customers and facilities, and operational or capital costs. The quality of the service is not taken into account, despite the recent investigations into the patient's choice of the hospital, which reveals that patients are willing to travel to reach a better facility (e.g. [17], [19]). The reason for the absence of the quality criterion in the models is that the quality of a health care facility is hard to quantify. Usually, hospitals are classified into three levels, according to the ranges of technologies and sub-services. The lower level hospitals are denoted as primary [14] or sub-acute [1], the middle level hospitals are secondary, and the higher level is denoted as tertiary. Unlike the common, three-level classification, in our model each hospital is assigned a quality coefficient ranging from 0.8 to 1.46, where higher coefficient means higher quality. This ranking is adopted from the evaluation of hospitals done by Dôvera [13]. The evaluation was published in September 2008. It is the only publicly available measure of the quality, although it covers not only the quality of the hospital but also the accessibility, staff's proficiency level, and technical equipment. The quality parameter represents 50% of the ranking. It measures the level and the outcomes of the health care provided by the hospital. The weight values of the accessibility, staff's proficiency level, and technical equipment are 20%, 20%, and 10% respectively. In our model we have simplified the ranking parameter, regarding it as a measure of the quality only. The other simplification in our model lays in ignoring the existing categories of hospitals. The health insurance company divided hospitals into four groups: (1) teaching hospitals, (2) general hospitals, (3) specialized hospitals (e.g. children or psychiatrics hospitals), and (4) other bed facilities. The ranking of a hospital made by Dôvera depends on what other hospitals there are in the same group. Our model only takes into account teaching hospitals and general hospitals, and considers their ranking coefficients as absolute values.

The second criterion – complexity – is closely related to quality. We have decided to use complexity as a separate criterion because it has often been mentioned as an important feature of a hospital in various discussions among health care managers. In addition, we have been using the complexity factor to model the ability of hospitals to provide urgent health care. Hospitals providing urgent health care need to have at least the following departments: surgery, orthopaedics or traumatology, internal medicine or cardiology, neurology, gynaecology and obstetrics, and paediatrics [12]. In our model the hospital complexity factor is the ratio between the number of its emergency departments and 6 – the desired number. It means that the complexity factor of a hospital with all of the 6 departments is 1, and the complexity factor of a hospital with just one emergency department is 0.167.

The accessibility criterion minimizes the time that patients have to spend travelling from their places of residence to hospitals.

The equity criterion means that the times patients spend travelling to hospitals should not differ too much, so that people living in border-line areas or areas with bad transportation infrastructure can reach hospitals within acceptable time periods, just like people living in other areas around the country can. This is important especially in such cases when urgent medical care is required.

1 Mathematical Programming Model

This section describes the problem more closely, and states a mathematical programming formulation.

In terms of location theory, we face a discrete network location problem [2] because the facilities to be located (hospitals), as well as the customers (cities and villages where potential patients live) are located at the nodes of transportation networks. The set of candidate locations (denoted by I in our mathematical model) consists of all current hospital locations. The goal is to decide where $p < |I|$ hospitals should be located. We suppose that p is known in advance. Later, in the part about computational experiments, we show how the sensitivity analysis of this parameter can help a decision maker to study the impact of less or more drastic reduction of the hospital network. As mentioned in the introduction, two characteristics of a hospital i have been taken into account: quality (denoted by q_i) and the emergency departments ratio (denoted by o_i).

The set of customers (denoted by J) consists of all municipalities. Each individual municipality j has b_j inhabitants (potential patients).

In the model there are two types of bivalent variables. Variables $y_i \in \{0, 1\}$ model the decision whether hospital i will be included in the network ($y_i = 1$) or not ($y_i = 0$). The other set of bivalent variables x_{ij} models the allocation of customers to open hospitals. We are using a simple hospital choice model assuming that patient's choice only depends on distance, i.e. that a patient always chooses the nearest hospital. Thus variable x_{ij} takes the value 1, if hospital i is the nearest one to municipality j. Otherwise, it takes the value 0. Then the term

$$\sum_{j \in J} x_{ij} \tag{1}$$

defines which municipalities form the attraction zone of hospital i, and

$$\sum_{j \in J} b_j x_{ij} \tag{2}$$

is the size of the attraction zone measured by the number of its inhabitants.

The first objective of the design is to achieve the highest possible quality of the hospital network, i.e. the goal is to maximise the quality of the selected hospitals:

$$\text{Maximise } f_1(y) = \sum_{i \in I} q_i y_i \tag{3}$$

The second objective is to keep in the network the hospitals that are able to provide complex urgent health care. The objective is formulated as follows:

$$\text{Maximise } f_2(y) = \sum_{i \in I} o_i y_i \tag{4}$$

The accessibility criterion can be expressed as the total amount of time potential patients spend travelling to the nearest hospital. The time depends on the transportation infrastructure and the means of transport the patients use. We assume potential patients would use the road network and individual transport to get to hospitals. Based on the road quality, each road segment belongs to a class within a finite classification system. Further on, an average speed of a vehicle movement can be associated to each of the road classes. Then, the estimation of the necessary transport time for each road segment can be obtained from the segment length and the average speed corresponding to the segment class. Using segment times, the accessibility time t_{ij} required to get from municipality j to hospital i is enumerated as the time length of the shortest path connecting i and j. Because the average speeds are not constant but depend on the vehicle in use, the accessibility time $t_{ij}(v)$ is a function of vector $v = \langle v_1, v_2, \ldots, v_r \rangle$ of the speeds which the vehicle does on the roads of class 1, 2, ..., r (notation taken from Janáček [4]). Using this notation, the accessibility objective can be stated as

$$\text{Minimise } f_3(x) = \sum_{i \in I} \sum_{j \in J} t_{ij}(v) b_j x_{ij} \tag{5}$$

The equity criterion was verbally formulated by the Ministry of Health Care [12]. The ministry stated that an emergency ambulance transporting a patient from a place of accident must reach a hospital in 30 minutes, no matter where the accident has taken place. In terms of location theory this means that every municipality should be covered by the service in the time limit $T^{max} = 30$ minutes. However, this is an unrealistic requirement, especially in the geographic conditions of the Slovak Republic, where a part of population live in small villages far away from hospitals. Therefore, some municipalities may get left uncovered in the design, although they still need to be served. An equitable distribution of hospitals can be reached as soon as the total travel time for uncovered inhabitants is minimised:

$$\text{Minimise } f_4(x) = \sum_{i \in I} \sum_{\substack{j \in J \\ t_{ij}(u) > T^{max}}} t_{ij}(u) b_j x_{ij} \tag{6}$$

Now, patients are transported by emergency vehicles therefore another speed vector u is used.

The criteria are in conflict. Criterion f_3 attracts hospitals to economic and administrative centres, where a lot of potential patients live. On the other hand, criterion f_4 pulls hospitals towards the edges of the country. The impact of criteria f_1 and f_2 is similar to that of criterion f_3 because large and good hospitals are usually placed in bigger cities.

To solve this multiple-criteria optimisation problem, several methods can be used. In our case, where the target values of criteria are not known, the most suitable is a scalarization method, which converts a multiple objective program (MOP) to a single objective program (SOP) by means of a real-valued scalarizing function of the objectives and vector parameters [3]. We decided to use the most well-known scalarization technique, which is the weighted sum approach. In this approach the resulting objective function is a linear combination of the MOP objective functions.

The resulting single objective location-allocation model can be written as follows:

$$\text{Maximise } w_1 f_1(y)/N_1 + w_2 f_2(y)/N_2 - w_3 f_3(x)/N_3 - w_4 f_4(x)/N_4 \quad (7)$$

$$\text{Subject to } \sum x_{ij} = 1 \text{ for } j \in J \quad (8)$$

$$x_{ij} \le y_i \text{ for } i \in I, j \in J \quad (9)$$

$$\sum_{i \in I} y_i = p \quad (10)$$

$$y_i \in \{0, 1\} \text{ for } i \in I \quad (11)$$

$$x_{ij} \in \{0, 1\} \text{ for } i \in I, j \in J \quad (12)$$

First we explain the constraints and then we will return to the objective function.

Constraints (8) ensure that each municipality is assigned to exactly one hospital. Constraints (9) are so called binding constraints which ensure that a municipality j is assigned only to an open hospital i. Constraint (10) puts the limit p on the number of open hospitals. Constraints (11) and (12) are obligatory constraints on bivalent variables.

Component w_k, $k = 1, ..., 4$ of a vector parameter $\boldsymbol{w}$ in the objective function (7) is the weight which expresses the importance of the k^{th} objective. If $w_k \geq 0$ for all $k = 1, ..., 4$ and $\sum_{k=1}^{4} w_k = 1$, then a set of compromise (noninferior) solutions can be generated by varying weights of the particular criteria. The decision maker can then evaluate alternative solutions, possibly considering other quantitative criteria such as maximum and average travel distances, as well as unquantified factors such as the specialization of hospitals.

Component N_k, $k = 1, ..., 4$ of a vector parameter $\boldsymbol{N}$ in the objective function (7) is a normalizing factor which converts the objective values, so they are similar and close to one [7]. The normalizing factors can be defined as coordinates of the utopia point in the objective space. The utopia point in our problem corresponds to the complete hospital network. Closing some hospitals, all the objectives get worse. If $i(\boldsymbol{v}, j)$ stands for the nearest hospital time-wise to municipality j, considering the speeds given by $\boldsymbol{v}$, then the components of vector $\boldsymbol{N}$ can be defined as:

$$N_1 = \sum_{i \in I} q_i \quad (13)$$

$$N_2 = \sum_{i \in I} o_i \quad (14)$$

$$N_3 = \sum_{j \in J} b_j t_{i(\boldsymbol{v},j),j}(\boldsymbol{v}) \quad (15)$$

$$N_4 = \sum_{t_{j(\boldsymbol{u},j),j}(\boldsymbol{u}) > T^{max}} b_j t_{i(\boldsymbol{u},j),j}(\boldsymbol{u}) \quad (16)$$

2 Case Study

In the case study we have implemented the model in the Slovak Republic conditions. In this section we present two analysis aimed at (1) the impact of the reduction of the hospital network on the accessibility, and (2) comparison of mathematical solutions with the current networks of hospitals which have contracts with Dôvera and VšZP .

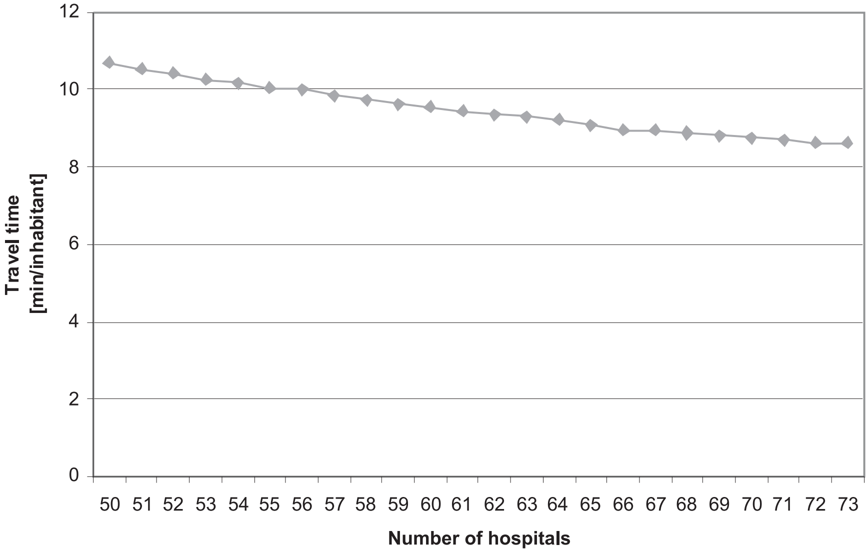
The input parameters of the model are:

$\lvert I \rvert = 73$	Maximum number of hospitals.
$\lvert J \rvert = 2916$	Number of municipalities.
$\boldsymbol{v} = \langle 85, 75, 55, 40, 30 \rangle$	Average speeds for individual transport in kilometres per hour on highways, roads of first, second, and third class, and on the local roads respectively (adopted from [11]).
$\boldsymbol{u} = \langle 105, 95, 75, 60, 50 \rangle$	Average speeds for emergency vehicles.
$\boldsymbol{w} = \langle 0.7, 0.1, 0.1, 0.1 \rangle$	Weights of the objectives.

The mathematical model was implemented in the general optimisation software Xpress-MP. The solution takes about 30 seconds on a personal computer equipped with the Intel Core 2 6700 processor with 2.66 GHz and 3 GB of RAM.

The impact of the hospital network reduction on the transportation accessibility was investigated by sensitivity analysis on the mathematical model

Fig. 1: The impact of the reduced number of hospitals on the transportation accessibility



Source: own

(7)-(12). The sensitivity of the solution, particularly the value of the objective function f3, to the number of hospitals p was analysed. Parameter p was decreasing from 73 to 50. As mentioned before, f_3 expresses the transportation accessibility of hospitals as the total travel time potential patients spend on their way to the nearest hospital, assuming they use a car. Because the total travel time is a high figure, clearer notion of the accessibility can be obtained by dividing the value of f_3 by the total number of inhabitants, giving the average travel time per one inhabitant.

The dependency between the accessibility and the number of hospitals is presented in Fig. 1. The graph confirms the expected trend that the accessibility is failing with the decreasing number of hospitals. In the complete network of 73 hospitals, the average travel time is 8.6 minutes per inhabitant, while reducing the network to 50 hospitals makes the average travel time increase to 10.7 minutes per inhabitant .

Table 1 compares the complete hospital network and four reduced networks in terms of the number of open hospitals, the quality of the hos-

Tab. 1: Optimisation vs. reality

Hospital network	Number of hospitals	Sum of the quality coefficients [-]	Average travel time [minutes/ inhabitant]	Uncovered population [inhabitants]
December 2007	73	74.72	8.62	39593
Dôvera, September 2008	65	68.03	9.35	52585
Mathematical solution	65	67.82	9.08	39593
VšZP, September 2008	69	70.79	8.82	39593
Mathematical solution	69	71.28	8.81	39593

Source: own

pitals in the network, the average travel time, and the uncovered population, i.e. the number of inhabitants living behind the 30 minutes limit from the nearest hospital. The complete network (as was at the end of 2007) is evaluated in the first row. In the second and third row there are comparisons of the network reduced by Dôvera and the optimised network with the same number of hospitals (with parameter p set to 65). Compared to the health insurance company design, the mathematical model emphasises the accessibility of hospitals (criterion f_3) and the equity in the access to health care (criterion f_4). That is why the accessibility for regular patients as well as for patients transported to hospitals by the emergency medical service is better in the network proposed by the mathematical model, and, at the same time, the total quality is worse. The reduction performed by VšZP is not so dramatic. The VšZP network and the mathematical design (the fourth and fifth row of the table) differ only in case of one hospital. As we can see, this difference has no impact on the accessibility. On the other hand, VšZP has chosen worse hospital than the mathematical model has. The possible reason may be that the quality coefficients were set by Dôvera, and they need not reflect the criteria of other health insurance companies.

Conclusion

The paper describes an analytical approach to the problem of reducing the hospital network. The proposed mathematical model can serve as a managerial tool in an effort to improve the existing health care system. The model takes into account interaction between the conflicting objectives. Changing the weights of the objectives, a set of the alternative solutions may be generated and evaluated. This way a planner can investigate the various scenarios of the health sector reform. The proposed model does not need extensive data. Because of a relatively small number of possible locations, it may be solved in a reasonable time.

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ABSTRACT

REDUCTION OF A HOSPITAL NETWORK AS A MULTIPLE CRITERIA OPTIMISATION PROBLEM

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The paper describes the design of a hospital network using mathematical programming approach. The problem is formulated as a multiple-criteria location problem. The criteria reflect patients' demand for high-quality and accessible health care. It means that the network should include: (1) high-quality and (2) complex hospitals with a broad range of emergency departments, (3) ensuring good transportation accessibility for most people, and at the same time (4) equitable distributed across the region. The economic criterion emanating from a limited budget is implemented as a constraint on the number of open hospitals. Given a complete network of hospitals and the desired number of hospitals, the model decides (according to the multiple criteria) which hospitals should be closed down. At the same time, it determines the allocation of municipalities to the open hospitals, defining the attraction zone of each hospital. Changing the weights of the objectives, a set of the alternative solutions (hospital networks) may be generated and evaluated. Thus the model can be used as a managerial tool in the process of reducing the hospital network with the aim of increasing the efficiency of the provided medical care. In the case study we have implemented the model in the Slovak Republic conditions. We present two analyses aimed at (1) the impact of reducing the hospital network on the accessibility, and (2) comparison of the mathematical solutions with the current networks of hospitals which have contracts with two biggest health insurance companies.

Key Words: Optimization Techniques, Programming Models, Analysis of Health Care Markets.

JEL Classification: C61, I11.