ERLANDER PRINCIPLE IN MANAGERIAL DECISION MAKING ON CZECH AND SLOVAK URBAN TRANSPORT ROUTES

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Introduction

The first urban public transport lines were established separately, on the basis of individual local needs, in the first half of the 19th century. They used horse-drawn omnibuses. An interesting coincidence is that the first public omnibus line was put into operation both in Prague and in London in 1829.

A few decades later, there was promoted the effort to merge the individual lines into coherent systems under common management. For instance, the London General Omnibus Company (LGOC) was founded in 1855. Karlin Omnibus Company (Karlin is a quarter of Prague) arose in 1870. Routes and frequencies of these systems were created intuitively and were modified based on experience. The same become true when the omnibuses were motorized.

The first efforts to create the routes and their frequencies objectively, justified by some engineering experience and knowledge, appeared after the World War I. One can mention the paper [23]. However, "manual" techniques were not able to handle the huge number of all possible lines, less so all their possible configurations. Any mathematically based method, intended to design bus routes and frequencies, could have, generally speaking, the character of a simple, hand-feasible, heuristics. There is, however, one exception. Creation of parallel routes together with their frequencies on small size square grid networks can be done by conventional analytical methods even manually, see [4]. Here should be noted that throughout this article, 'network' means an undirected graph $G = (V, E)$ where V is the set of vertices and E is the set of edges.

The change was brought by the possibility of using computers in the sixties of last century as documented in [19] or [21]. The use of mathematical methods and their implementation on computers, however, began to develop massively in the seventies, as shown in the survey paper [14] where more than 50 such papers are cited.

Computers were mostly used in "man-machine" mode. Predominant variant was the following: The person, i.e. the transport manager, proposes several configurations of routes with their frequencies and the machine configuration. The machine i.e. the computer, evaluate them. One such methodology was described in [22]. Other one, used e.g. in Sweden, is presented by [2].

Other authors, e.g. [29], used the following approach: First, a set R_0 of "candidate" routes is created, manually by a manager or with help of computer. Second, the configuration $R \subset R_0$ (to be operated) is selected. Third, optimal frequencies for the routes from R are determined.

The Swedish-Czechoslovak Symposium on Applied Mathematics was in Prague in March 1975. Sven Erlander, the well known Swedish expert, spoke about an original approach to urban bus routing and frequencing published in [17]. It consists of the following steps, where the originality is in the step S2:

S0: Assemblage of initial data on passenger demand, bus fleet and road network.

S1: Creation of the set R_0 containing reasonable bus routes. This step was usually done manually by a transport manager.

S2: Assignment an integer variable $x_r \geq 0$ to each route $r \in R_0$. The meaning of it is that the value x_r expresses the number of buses assigned to the route r. This variable accomplishes the selection of routes from the initial 'wide' set R_0 into (usually much smaller) set R which is to be put into operation. The value $x_r > 0$ means that the route r will operate x_r vehicles, $x_r = 0$ means that the route r is abandoned.

S3: Choice of all values x_r meeting the constraints on the bus fleet and minimizing the time lost by the passengers. This step is done by computer via non-linear programming.

The step S2 immediately woke overall interest, because the variable x_r did not fulfill only the standard selection role: $x_r = 1 \Rightarrow$ element r is selected, $x_r = 0 \Rightarrow$ element r is rejected. In the Erlander's model, meaning of the relation $x_r = 0$ remains unchanged, while $x_r \neq 0$ not only means acceptance of the route r into the operation, but it expresses the number of operating vehicles. Since that time, the step S2 is known as 'Erlander principle' to Czech and Slovak experts. It enables the simultaneous choice of the set of routes $R \subset R_0$ and their frequencies.

The first author of the current paper followed the lecture of S. Erlander. He realized that it was not possible to take this model unchanged in the Czech and Slovak conditions. The main reason was the non-linearity of it.

It is inevitable that the model is nonlinear, when the variable x_r expresses the number of buses assigned to the route r and the objective function expresses the delays of passengers. The reason is that the average waiting time for the next bus is of the form $const/x_r$, since the greater the number of buses x_r, the less the interval between successive vehicles and, consequently, the waiting time as well.

Moreover, there might happen that $x_r = 0$ in the denominator of the objective function, which is not permitted. Therefore, the model must be protected against it i.e. it should be even more complicated.

Such a variant of non-linear optimization problem is much more demanding on computer memory and calculation time than the linear one. In late seventies, the maximum size of non-linear problem, solved by Erlander model on Czechoslovak computers, was estimated to 15 routes in the set R0. However, the practical demand was to solve problems 10 or more times greater. Therefore the main question was whether there was some linear modification of the model.

1. Linear Modification of Erlander Model (LMEM)

If one wanted to keep the genial idea of the step S2 but to avoid the non-linearity of the problem, it was necessary to replace the indicator of passenger time loss by another one. However, the time loss of passengers is extremely important indicator. Any new objective function should not counteract it.

It is supposed that the passenger demand is represented by an origin-destination matrix (briefly OD-matrix), i.e. the square matrix having n rows (and n columns as well) where n is the number of nodes (usually stops of urban transport). The element f_{ij} in the i^{th} row and j^{th} column of the OD-matrix F expresses the passenger flow, i.e. the number of passengers (an hour) demanding the transport from the node i to the node j. Usually, there are given several OD-matrices, one for the morning peak another for the morning saddle, another for afternoon peak etc.

A new linear objective function not only must not counteract the reduction of time losses, but even it is desirable to support it. Czechoslovak specialists decided to choose the new indicator, expressing the number of seats over the number of demanding passengers. They found out that there were two possibilities: either to take the new indicator in a constraint and to minimize the necessary number of vehicles, as described in [11] and [12], or to leave the number of vehicles in the constraint and to maximize new indicator, see e.g. [13]. In both cases it is supposed that all passengers, counted in the OD-matrix, are transported without exceeding the bus capacity anywhere in the network. It is obvious that if each passenger is transported and, despite this, the new indicator remains small, then the duration of the trips, as a part of passenger time losses, ought to be small as well. The new and the old indicators are conformable.

1.1 The Linear Model Minimizing the Fleet

It is assumed that, after the traffic assignment, for each edge e of the network G and for both its direction, it is known the total number of passengers per hour who want to ride through this edge in the direction. The number f_e denotes the greatest of these values.

It is also assumed that a set of candidate routes R_0 is prepared and the following data are given for each $r \in R_0$:

E_r – the set of all edges passed by the route r,

t_r – [in minutes] the cycle time of vehicles on the route, i.e. the round trip running time plus layover time at each end, *c* – capacity of each bus.

Problem P1: Let the network *G* and the candidate set R_0 be given. The problem is to find a nonnegative integer x_r for each $r \in R_0$ such that

$$z = \sum_{r \in R_0} x_r \to \min , \tag{1}$$

$$\sum_{r:e \in E_r} \frac{60c}{t_r} x_r \geq f_e \quad \text{for each } e \in E \tag{2}$$

Commentary: In the constraint (2), the fraction $60/t_r$ represents the frequency, i.e. the number of buses (per hour) on the route *r* if exactly one vehicle is assigned to it. Therefore $60c/t_r$ expresses the supply of places for passengers per hour by one vehicle assigned to *r* and, $(60c/tr)x_r$ is the supply if x_r vehicles are assigned. Finally, the sum on the left side of the constraint (2) represents the total number of places in vehicles of all lines passing through the edge *e*. Condition (2) then requires that this offer is not less than the demand f_e.

Remark: As said in [11] concerning the problem **P1**, it may happen that the resulting percentage of indirectly traveling passengers (i.e. with at least one transfer) is too high, e.g. they exceed 15 %. Then, in order to reduce it, it is possible to look for such elements f_{ij} of the OD-matrix *F*, that the set $R_{ij} \cap R = \varnothing$ where $R_{ij} = \{r \in R_0: r$ contains both vertices $i, j\}$. If such elements exist then the maximal of them is chosen. Let it be f_{vw}. The following constraint is added to the problem **P1**:

$$\sum_{r \in R_{vw}} x_r \geq 1 \tag{3}$$

Then, at least one route from the new resulting set *R* contains both *v* and *w*. I.e. a new possibility of direct travel from the vertex *v* to *w* arose for f_{vw} passengers per hour. Consequently, the number of indirectly traveling passengers decreases. If it is not yet satisfactory, another constraint of the type (3) can be added etc.

1.2 The Linear Model Maximizing the Comfort of Passengers

The second author of the current article joined the research team in 1984. She noticed that managers of Czech and Slovak transport companies together with the ones of municipal administration did not feel the lack of vehicles as the main problem. Consequently, the minimization of number of needed vehicles is not the objective of the highest importance for them. They have available rolling stock with drivers and they felt no problem with letting them work. Their main problem was overcrowding of vehicles. E.g. it happened in the town of Olomouc in late eighties that some trams transported twice as many passengers then their official capacity was.

Since the overcrowding is unacceptable at any segment of the network, the objective is of the mini-max type. I.e. the maximal overcrowding taken from the set of all edges of the network ought to be minimized. However, such a formulation does not lead to linear function of the variables x_r. Therefore, it was replaced by an equivalent objective – to maximize the minimal relative reserve of places in vehicles as described in [7]. The model and the method were called PRIVOL there. It is a Slovak acronym of "PRIdelenie VOzidiel na Linky" (assignment of vehicles to routes).

A similar type of objective function was chosen by Han and Wilson [18]. They minimized the maximum occupancy level of vehicles for each route.

As mentioned above, $s_r = (60c/t_r)x_r$ expresses the supply of places per hour if x_r vehicles are assigned to the route *r*. It is valid for each edge the route *r* is passing through. The relative reserve of places at the edge e equals to the fraction

$$\frac{\sum_{r \in R_e} \frac{60c}{t_r} x_r}{f_e} \tag{4}$$

where R_e denotes all routes $r \in R_0$ passing through the edge *e*. On this basis one can formulate the following problem:

Problem P2: Let the network *G* and the candidate set R_0 be given. Let *n* be the number of available vehicles. The problem is to find a positive real number *y* and a nonnegative integer x_r for each $r \in R_0$ such that

$$y \to \max \tag{5}$$

$$\sum_{r \in R_0} x_r = n \tag{6}$$

$$\sum_{r:e \in E_r} \frac{60c}{t_r} x_r \geq f_e y \quad \text{for each } e \in E \tag{7}$$

Remark: The constraint (3) can be added similarly as at the problem **P1** in order to decrease the unacceptably high percentage of indirectly traveling passengers.

1.3 Modifications of the Problem P2

As seen in [13], the problem **P2** can be modified. One can note that similar modification is able for **P1** as well.

The modification is focused on the case of **multiple system urban transport**. It happens e.g. when three modes, i.e. tram, trolleybus and bus, are operating on surface level in a town. Then the set R_0 of candidate routes is divided into three disjoint subsets R_{01} (candidates for tram routes), R_{02} (candidates for trolleybus routes) and R_{03} (candidates for bus routes). The problem **P2** which is defined by (5), (6) and (7), is modified to **P2a** by replacing the constraint (6) by the constraint

$$\sum_{r \in R_{0k}} x_r = n_k \quad \text{for } k = 1,2,3 \tag{8}$$

where n_1, n_2, n_3 means the available number of tram units, trolleybuses and buses respectively.

Other modification consists of changing the meaning of the variable x_r. Instead of number of vehicles assigned to the route r, the modified meaning of x_r is dynamic capacity, i.e. the number of places for passengers per hour. It is easy to show that this modification represents only a linear transformation of variables.

2. LMEM in Czech and Slovak Republic in the Last 20 Years

The urban transport routing and frequencing problem was studied quite intensively around the world in that period. One can mention e.g. the survey papers [16], [20] and [28], citing 20, 60 and 70 papers on routing and frequencing from the last twenty years respectively. Among them, one can find many interesting articles, e.g. [1] focused on the use of genetic algorithm, [5] presenting a three-phases heuristic procedure, which was successfully applied in Rome, [6] combines mathematical programming approaches with decision-making and, last but not least, [3] presents a very successful column generation approach to routing and frequencing.

2.1 Theoretical Results of Czech and Slovak Authors

None of the cited papers is of Czech or Slovak origins. Nevertheless, new research results and their practical applications arose in this pair of countries as well.

The preparatory phase of the routing process, i.e. the choice of the basic network G, is studied in [8], the paper having two Czech and two Slovak authors. The main optimization problem there is to find the cheapest subnetwork G of a given wider "candidate" network G_0 such that the distance $dG(v, w)$ of any important pair of vertices v, w on G is not greater than $qdG_0(v, w)$ where $dG_0(v, w)$ is the distance of v and w on G_0 and $q \geq 1$ is a given number.

From Czech origin, one can mention e.g. the paper [15] where four different models, representing four modifications of the problem **P1**, are examined. Moreover, a number of numerical experiments concerning the network corresponding to a medium-sized town were performed using the four constructed models. These numerical experiments demonstrated the functionality of the designed models.

The book [9] is from Czech origin as well. There the subchapter 12.3 is focused on routing and frequencing. There the problem **P2** and its solution is presented for several modifications. It is said, what happens if

- the number x_r of assigned vehicles is allowed to be non-integer,
- the symbol x_r does not mean the number of vehicles, but the total places for passengers per hour.

Moreover, in [9] one can find the square-root rule concerning the relation between results of optimization, if the objective of maximal comfort of travelling is replaced by minimization of passenger time loss while waiting for the bus. It is shown that if t_r denotes the vehicle cycle time on the route r and p_r is the number of passengers per hour transported by the route r, then the numbers of vehicles allocated to individual lines are in proportion

- $p_1t_1 : p_2t_2$: ... if the objective is the relative reserve of places for passengers,
- $\sqrt{p_1t_1} : \sqrt{p_2t_2}$: ... if the objective is the time loss while waiting for the bus.

Among Czech made papers one can mention also [10] presenting original results concerning optimal routing in small demand areas. The problem the paper deals with is the following:

Let $G = (V, E, q, d)$ be a (non-oriented) graph representing the network suitable for walking. Let q be a demand function $q: V \rightarrow \langle 0; \infty)$ and let d be a length $d: E \rightarrow (0; \infty)$. Let $d(u, v)$ be the distance of $u, v \in V$ and $d(u, S) = \min\{d(u, v): v \in S\}$ be the distance of the vertex $u \in V$ from the set $S \subset V$. Let $W \subset V$ and $GW = (W, F, \delta)$ be a graph suitable for bus transit with edge length δ (not necessarily equal to d on $E \cap F$). Let $\delta(S)$ be the length of the shortest path containing the vertices of S in GW for every $S \subset W$ (the set S represent "candidates" for bus stops, the path represents a possible bus route connecting S). Let $\lambda \in (0; \infty)$ represents the accessibility limit (e.g. $\lambda = 0.3\ km$) and $q = \sum_{v \in V} q(v)$. The problem is to find $S \subset W$ such that for the average walking distance to the closest bus stop $\mu(S)$

$$\mu(S) = \frac{1}{q} \sum_{v \in V} q(v) d(v, S) \leq \lambda, \qquad (9)$$

$$\delta(S) \rightarrow \min \qquad (10)$$

The paper [27] presents the author's experience with computer programs for heuristic and exact solution of the problem (9), (10).

The most important Slovak author dealing with routing and frequencing is Peško. In the paper [24] Peško seeks a circular route passing through all demand points, while in [10] a reduction of this set is admitted. In the [25], Peško allows refusing a part of demand but [10] does not.

From the viewpoint of routing and frequencing [26] brings several original results concerning the problem **P2**. First, Peško changed both objective function and the definition of the variable y. He replaced the meaning of the minimum relative reserve y by the absolute reserve. Second, he studied the relaxation of the problem omitting the constraint that the number of buses assignet to a route must be integer. Third another relaxation is obtained by omitting the constraint that each passenger must be transported.

2.2 Practical Applications in Czech and Slovak Towns

The authors of the current paper noticed about 10 applications of PRIVOL in Czech and Slovak Towns. Somewhere the set of routes was changed almost completely, e. g. in Žilina (Slovakia – in that time about 90 thousands inhabitants). Formerly there were 33 routes with quite small frequencies. The new system resulting from PRIVOL had only 14 routes whereas only 3 routes remained unchanged. Since the rolling stock remained unchanged the frequencies increased considerably. Similarly, the substantial changes were implemented in Piešťany (Slovakia – population about 40 thousands). Other towns changed only small part of their routes, e.g. Pardubice (Czech Republic – about 90 thousands). Only 3 from 16 routes were changed.

3. Expected Future Development

The authors of the current paper expect that modifications of the models and methods, derived from the Erlander principle and from the PRIVOL will appear in future as well. One can presuppose that the Czech and Slovak authors mentioned in the preceding paragraph will continue their research work at least. In the theory still remain open such problems like

- incorporation of number of transfers into the objective function,
- incorporation of demand elasticity with respect to the transport supply into the constraints.

Conclusion

The paper outlines development of the routing and frequencing problem history as a part of the history of public transport, as a consequence of the fact, that public transport became a mass need of people. It is shown how complex problem it is and, consequently, that an objectively acceptable solution can be obtained only by computer. It is documented by the fact that large number of citations appeared simultaneously with the mass deployment of personal computers.

The core of the article is a description of how the Erlander approach to routing and frequencing was applied in the former Czechoslovakia and after the division into two independent states. The paper describes the models and methods that are very successfully applied in practice and cited the papers presenting them. Finally, the expectations of future development are outlined.

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Abstract

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In the beginning of the paper, the position of routing problems in the historical development of public transport is outlined. It is explained why the boom of research papers on this issue came in the seventies of the last century. It is described, how the contacts of Czechoslovak and Swedish researchers, at the same time, motivated the first ones to use the Erlander principle, i.e. starting with a set of "candidate" routes and afterwards choosing some of them by variables which are equal to the number of assigned buses.

Afterwards it is shown that the original Erlander approach used the total passenger time losses as objective function. It led to a non-linear model since the unknown variable expressing the number of vehicles enters into the denominator of the objective function. Another complication of the model was in the necessity to avoid zeroes in the denominators. Czechoslovak researchers did not possess any computer enabling to solve non-linear optimization problems of the dimensions met in practice, Therefore, they decided to replace the indicator of total passenger time losses by another one which would not contain the decision variable in the denominator, but in the numerator. They took the numerical ratio of number of places in vehicles to the number of passengers.

Further, the paper embodies the results achieved in the Czech and Slovak Republic into the context of world literature. The optimization methodology PRIVOL is then described in details, together with a brief outline of its application.

Finally, the further development of research is predicted. E.g. the incorporation of demand elasticity with respect to the transport supply into the constraints in the model.

Key Words: *manager; decision, transport, route, frequency, method.*

JEL Classification: *R42, O18, C61.*